# A New BISON-like Construction Block Cipher: DBISON 

Haixia Zhao ${ }^{1,3}$, Yongzhuang Wei ${ }^{2 *}$, and Zhenghong Liu ${ }^{1}$<br>${ }^{1}$ Key Laboratory of Cognitive Radio and Information Processing, Ministry of Education, Guilin University of Electronic Technology, Guilin, 541004, China [e-mail: guetzhx@163.com, giet.liu@163.com]<br>${ }^{2}$ Guangxi Key Laboratory of Cryptography and Information Security, Guilin University of Electronic Technology, Guilin, 541004, China<br>[e-mail: walker_wyz@guet.edu.cn]<br>${ }^{3}$ School of Mathematics and Computing Science, Guilin University of Electronic Technology, Guilin 541004, China<br>*Corresponding author: Yongzhuang Wei

Received April 25, 2021; revised January 26, 2022; accepted May 11, 2022;
published May 31, 2022


#### Abstract

At EUROCRYPT 2019, a new block cipher algorithm called BISON was proposed by Canteaut et al. which uses a novel structure named as Whitened Swap-Or-Not (WSN). Unlike the traditional wide trail strategy, the differential and linear properties of this algorithm can be easily determined. However, the encryption speed of the BISON algorithm is quite low due to a large number of iterative rounds needed to ensure certain security margins. Commonly, denoting by $n$ is the data block length, this design requires $3 n$ encryption rounds. Moreover, the block size $n$ of BISON is always odd, which is not convenient for operations performed on a byte level. In order to overcome these issues, we propose a new block cipher, named DBISON, which more efficiently employs the ideas of double layers typical to the BISON-like construction. More precisely, DBISON divides the input into two parts of size $n / 2$ bits and performs the round computations in parallel, which leads to an increased encryption speed. In particular, the data block length $n$ of DBISON can be even, which gives certain additional implementation benefits over BISON. Furthermore, the resistance of DBISON against differential and linear attacks is also investigated. It is shown the maximal differential probability (MDP) is $1 / 2^{n-1}$ for $n$ encryption rounds and that the maximal linear probability (MLP) is strictly less than $1 / 2^{n-1}$ when $(n / 2+3)$ iterative encryption rounds are used. These estimates are very close to the ideal values when $n$ is close to 256.


Keywords: BISON block cipher, DBISON block cipher, Differential cryptanalysis, Linear cryptanalysis, WSN construction.

[^0]
## 1. Introduction

Block ciphers play an important role in the area of data storage and secure transmission in an open internet environment. During the past three decades, block ciphers have received a lot of attention from academic and industrial community.

Generally, security and implementation efficiency can be considered as the most crucial aspects in the design of block ciphers. To achieve sufficient security margins, block ciphers commonly employ multiple encryption rounds for the purpose of achieving a satisfactory level of diffusion and confusion [1]. On the other hand, the internal structure of a block cipher is also importance since it directly affects the implementation cost and performance in both hardware and software. Currently, the most prominent block ciphers employ diverse structures such as Feistel [2, 3], SPN [4], MISTY [5] and Lai-Massey [6], among others. A very common approach is to implement a block cipher as a substitution permutation network (SPN), which was extensively used in many prominent block ciphers, including AES [4, 7] whose design additionally embeds the concept of wide trail strategy[8]. One important issue with this design rationale regards the problem of determining the differential or linear properties of a given cipher, which is considered to be quite a difficult task. More specifically, in order to ensure good resistance against differential and linear cryptanalysis, the so-called branch number of diffusion (linear) layer and the cryptographic properties of the S-boxes (used in the substitution layer) have to be taken into account [9, 10]. Due to the iterative structure of block ciphers and an exponential growth of possible differential/linear patterns, the exact security estimates are not easy to specify. An alternative design rationale of constructing block ciphers that achieve an optimal security level (under the ideal model assumption) was introduced in [11]. This method uses the so-called Whitened Swap-Or-Not (WSN) construction, which itself is based on the Swap-or-Not method introduced in [12] and applicable in the settings when the internal functions are kept secret. Furthermore, instead of the need for a set of random Boolean functions for the Swap-or-Not method, the WSN approach [11] requires only two public random $n$-variable Boolean functions to achieve full security. Actually, there are very few known instances of WSN and an encryption algorithm based on this approach was specified in [12] but later broken by Vaudenay [13]. Another example of using the WSN method is the BISON block cipher, which was proposed by Canteaut et al. at EUROCRYPT 2019 [14]. The design of BISON implements XOR-ing of the round keys by using a quadratic bent function. Additionally, BISON seems to be resistant against differential cryptanalysis [15], linear cryptanalysis [16], and algebraic cryptanalysis [17] provided that the number of rounds is approximately $3 n$, where $n$ is the data block length and $n$ is odd. In particular, the MDP value of BISON can be easily evaluated without the exact details about its components, which is completely different to the wide trail strategy.

Consequently, the encryption speed of BISON is quite low due to a large number of rounds used and a large $n$-bit input size. For instance, assuming that $n=127$ implies that there are 381 rounds and additionally one needs to implement a large 126-bit nonlinear function which is quite demanding. To overcome these issues, we propose a new block cipher that borrows the design ideas from BISON, named DBISON. More specifically, the length of data block of DBISON is even and therefore the input $x$ can be divided into two halves $x_{L}$ and $x_{R}$ which are then processed in parallel using a similar structure as in Feistel networks. The details of round operations are given in Fig. 1 and, additionally, the used parameters are
described in Definition 4. Notice that, to complete the round operation, the left and right branch are swapped but in the final round the swap operation is not performed.


Fig. 1. The round function of DBISON
It will be shown that DBISON is resistant against both differential and linear cryptanalysis when the number of rounds $r$ reaches $n$. More specifically, we show that the MDP value equals $1 / 2^{n-1}$ when $n$ encryption rounds are used, whereas the MLP is strictly less than $1 / 2^{n-1}$ if at least $(n / 2+3)$ encryption rounds are applied. It is worth mentioning that the MDP can almost reach the ideal value $1 / 2^{n}$ if the size of data block $n$ is close to 256 . A comparison between BISON and DBISON is given in Table 1. However, to ensure that the algebraic degree of DBISON attains its maximal value $n$, the number of rounds is approximately $3 n$. DBISON offers a significant advantage over BISON in terms of encryption/decryption speed since the input size is divided into two halves (each having $n / 2$ bits) which are processed in parallel.

The rest of this paper are organized as follows. In Section 2, the DBISON block cipher is fully described. In Section 3, the differential cryptanalysis against DBISON is examined and the estimates of its MDP are provided. In Section 4, the resistance of DBISON against linear cryptanalysis is analyzed and the bounds on its MLP are derived. In Section 5, certain specific instances of DBISON are specified. Some concluding remarks can be found in Section 6.

Table 1. Comparison of BISON and DBISON

| Algorithm | Nonlinear function | MDP | MLP | Source |
| :--- | :---: | :---: | :---: | :---: |
| BISON | $n$-bit input size | $2^{-(n-1)}$ <br> $(n$-round) | $2^{-(n-1)}$ <br> $(n$-round) | $[14]$ |
| DBISON | Two $n / 2$-bit input halves <br> processed in parallel | $\leq 2^{-(n-1)}$ <br> $(n$-round) | $<2^{-(n-1)}$ <br> $((n / 2+3)$-round) | New |

## 2. Preliminaries

Definition $1{ }^{[18]}$ Let $F$ be a function from $F_{2}^{n}$ into $F_{2}^{n}$. For any $u, v \in F_{2}^{n}$, define $W_{F}(u, v)=\sum_{x \in F_{2}^{n}}(-1)^{u \bullet \otimes \oplus v F(x)}$, where - denotes the inner product in $F_{2}^{n}$, that is $u \bullet x=u_{1} x_{1} \oplus u_{2} x_{2} \oplus \ldots \oplus u_{n} x_{n}$. The multiset $\left\{W_{F}(u, v) \mid u, v \in F_{2}^{n}\right\}$ is called the Walsh
spectrum of $F$.
Definition $2{ }^{[19]}$ The $r$-round differential characteristic of an iterative block cipher is denoted as $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right)$. Assuming that the round keys $k_{1}, k_{2}, \ldots, k_{r}$ are independent and uniform, the differential characteristic probability $\operatorname{DP}(\Omega)$ is defined as $\mathrm{DP}(\Omega)=\prod_{i=1}^{r} \mathrm{DP}\left(\delta_{i-1}, \delta_{i}\right)$, i.e. it is the probability that the difference between input pair is $\delta_{0}$ and the difference between intermediate state $\left(y_{i}, y_{i}^{*}\right)$ is $\delta_{i}, 1 \leq i \leq r$.

Definition $3{ }^{[19]}$ The $r$-round linear characteristic of an iterative block cipher is denoted as $\theta=\left(\theta_{0}, \theta_{1}, \ldots, \theta_{r}\right)$. Assuming that the round keys $k_{1}, k_{2}, \ldots, k_{r}$ are independent and uniform, the linear characteristic probability $\operatorname{LP}(\theta)$ is defined by $\operatorname{LP}(\theta)=\prod_{i=1}^{r} \operatorname{LP}\left(\theta_{i-1}, \theta_{i}\right)$, i.e. the probability that the input mask is $\theta_{0}$ and the mask of an intermediate state $y_{i}$ is $\theta_{i}, 1 \leq i \leq r$.

For the input and output difference $(\alpha, \beta)$, it is a difficult task to compute the MDP of $(\alpha, \beta)$, even for a small number of rounds. However, computing the MDP of an $r$-round differential characteristic $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right)$ is an easier task, and the MDP of $\Omega$ also reflects the ability of the cipher to resist differential cryptanalysis. A similar reasoning applies when the MLP values is considered, thus having an initial mask $(a, b)$ and an $r$-round linear trail $\theta=\left(\theta_{0}, \theta_{1}, \ldots, \theta_{r}\right)$. We will investigate in detail the properties of DBISON in this context, hence its resistance against differential and linear cryptanalysis by providing the estimates on MDP and MLP using $\Omega$ and $\theta$, respectively.

Definition 4 Let the data block length of DBISON be $n=4 m+2$, where $m$ is a positive integer. The input $x$ of any encryption round is divided into the left half and right half, i.e. $x=\left(x_{L}, x_{R}\right)$. The $i$-th round function $F_{k_{i}, w_{i}}(x): F_{2}^{n} \rightarrow F_{2}^{n}$ is defined as

$$
\begin{equation*}
F_{k_{i}, w_{i}}(x)=\left(x_{L} \oplus x_{R} \oplus f_{i R}\left(w_{i R} \oplus \Phi_{k_{i \mathbb{R}}}\left(x_{L} \oplus x_{R}\right)\right) k_{i R}, x_{L} \oplus f_{i L}\left(w_{i L} \oplus \Phi_{k_{i L}}\left(x_{L}\right)\right) k_{i L}\right), \tag{1}
\end{equation*}
$$

where $k_{i}=\left(k_{i L}, k_{i R}\right), w_{i}=\left(w_{i L}, w_{i R}\right)$ are round keys ( $w_{i}$ is the whitened key), and $f_{i L}$ and $f_{i R}$ are bent functions with $n / 2-1$ variables. Moreover, $\Phi_{k_{l l}}, \Phi_{k_{\mathbb{R}}}: F_{2}^{n / 2} \rightarrow F_{2}^{n / 2-1}$ are linear functions and $\operatorname{ker} \Phi_{k_{i L}}=\left\{0, k_{i L}\right\}$, $\operatorname{ker} \Phi_{k_{i R}}=\left\{0, k_{i R}\right\}$, where $k_{i L}$ and $k_{i R}$ are generated by two LFSRs so that $k_{i L} \neq 0$ and $k_{i R} \neq 0$, respectively.

Remark 1 The analysis in this work follows two basic assumptions of symmetric cryptanalysis, i.e. the whitened keys are linearly independent, and the round keys satisfy the so-called random equivalence hypothesis.

## 3. Differential cryptanalysis of DBISON block cipher

The derivative of a function $f$ in direction $\alpha$ is defined as $D_{\alpha} f(x)=f(x) \oplus f(x \oplus \alpha)$. A successful application of differential cryptanalysis against block ciphers heavily relies on the differential properties of its substitution layer. The round function $F$ of a block cipher with $n$-bit input and output can be viewed as a vectorial Boolean function $F: F_{2}^{n} \rightarrow F_{2}^{n}$. The behavior of the derivatives of $F$ are described by the Differential Distribution Table (DDT) of $F$, whose entries are

$$
\operatorname{DDT}_{F}[\alpha, \beta]=\left|\left\{x \in F_{2}^{n} \mid F(x) \oplus F(x \oplus \alpha)=\beta\right\}\right|,
$$

where $\alpha \in F_{2}^{n}$ is referred to as the input difference and $\beta \in F_{2}^{n}$ as the output difference.
In this context, we are primarily interested in the DDT of the round function $F_{k_{i}, w_{i}}(x)$, which can be calculated explicitly using Theorem 1 below.

Theorem 1 Using (1), the round function of DBISON can be rewritten as

$$
\begin{equation*}
F(x)=\left(x_{L} \oplus x_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right) k_{R}, x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right) . \tag{2}
\end{equation*}
$$

Then $\operatorname{DDT}_{F}[\alpha, \beta]$ can be specified as follows:

1) $\mathrm{DDT}_{F}[\alpha, \beta]=2^{n}$ if $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right)$ and $\alpha \in\left\{\mathbf{0},\left(\mathbf{0}, k_{R}\right),\left(k_{L}, k_{L}\right),\left(k_{L}, k_{L} \oplus k_{R}\right)\right\}$.
2) $\operatorname{DDT}_{F}[\alpha, \beta]=2^{n-1}$ if $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right)$ and $\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \in\left\{\left(\mathbf{0}, \alpha_{L}\right),\left(k_{R}, \alpha_{L}\right)\right.$,
$\left.\left(\alpha_{L} \oplus \alpha_{R}, \mathbf{0}\right),\left(\alpha_{L} \oplus \alpha_{R}, k_{L}\right) \mid \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\}, \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\}\right\}$, or $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus\left(\mathbf{0}, k_{L}\right)$, $\alpha_{L} \oplus \alpha_{R} \in\left\{\mathbf{0}, k_{R}\right\}$ and $\alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\}$, or $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus\left(k_{R}, \mathbf{0}\right), \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\}$ and $\alpha_{L} \in\left\{\mathbf{0}, k_{L}\right\}$.
3) $\operatorname{DDT}_{F}[\alpha, \beta]=2^{n-2}$ if $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \gamma, \gamma \in\left\{\mathbf{0},\left(\mathbf{0}, k_{L}\right),\left(k_{R}, \mathbf{0}\right),\left(k_{R}, k_{L}\right)\right\}$ and $\alpha_{L} \oplus \alpha_{R}$ $\notin\left\{\mathbf{0}, k_{R}\right\}, \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\}$.
4) Otherwise, $\operatorname{DDT}_{F}[\alpha, \beta]=0$.

Proof Using the definitions of DDT and $F(x), \operatorname{DDT}_{F}[\alpha, \beta]$ can be deduced as:

$$
\operatorname{DDT}_{F}[\alpha, \beta]
$$

$$
\begin{equation*}
=\left|\left\{x \in F_{2}^{n} \mid\left(D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right) k_{R}, D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right)=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta\right\}\right| \tag{3}
\end{equation*}
$$

Clearly, $\operatorname{DDT}_{F}[\alpha, \beta]=0$ if $\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta \notin\left\{\mathbf{0},\left(\mathbf{0}, k_{L}\right),\left(k_{R}, \mathbf{0}\right),\left(k_{R}, k_{L}\right)\right\}:=K^{*}$.
In the following, we split our analysis of $\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta$ into four cases.
Case 1. $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right)$.
By (3) and $k_{L} \neq \mathbf{0}, k_{R} \neq \mathbf{0}$, it can be deduced that
$\operatorname{DDT}_{F}[\alpha, \beta]=\mid\left\{x \in F_{2}^{n} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right)=0\right.$ and $\left.D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=0\right\} \mid$.
(1) $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right) \neq \mathbf{0}$ and $\Phi_{k_{L}}\left(\alpha_{L}\right) \neq \mathbf{0}$. Denote $w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)$ by $x_{L}^{\prime}$.

Since $f_{L}$ is a bent function, thus $\left|\left\{x_{L}^{\prime} \in F_{2}^{n / 2-1} \mid D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(x_{L}^{\prime}\right)=0\right\}\right|=2^{n / 2-2}$. Furthermore, $\Phi_{k_{L}}$ is a linear function from $F_{2}^{n / 2}$ to $F_{2}^{n / 2-1}$ and $\operatorname{ker} \Phi_{k_{L}}=\left\{\mathbf{0}, k_{L}\right\}$, and therefore $\left|A_{L}\right|:=\left|\left\{x_{L} \in F_{2}^{n / 2} \mid D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=0\right\}\right|=2^{n / 2-1}$. For any $a_{i} \in A_{L}, i=1,2, \ldots, 2^{n / 2-1}$, $\left|\left\{x_{R} \in F_{2}^{n / 2} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(a_{i} \oplus x_{R}\right)\right)=0\right\}\right|=2^{n / 2-1}$ since $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right) \neq \mathbf{0}$ and $f_{R}$ is a bent function. Therefore, $\operatorname{DDT}_{F}[\alpha, \beta]=2^{n / 2-1} \times 2^{n / 2-1}=2^{n-2}$.
(2) $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right) \neq \mathbf{0}$ and $\Phi_{k_{L}}\left(\alpha_{L}\right)=\mathbf{0} . \operatorname{DDT}_{F}[\alpha, \beta]=2^{n / 2} \times 2^{n / 2-1}=2^{n-1}$.
(3) $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)=\mathbf{0}$ and $\Phi_{k_{L}}\left(\alpha_{L}\right) \neq \mathbf{0} . \operatorname{DDT}_{F}[\alpha, \beta]=2^{n / 2-1} \times 2^{n / 2}=2^{n-1}$.
(4) $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)=\mathbf{0}$ and $\Phi_{k_{L}}\left(\alpha_{L}\right)=\mathbf{0} . \operatorname{DDT}_{F}[\alpha, \beta]=2^{n / 2} \times 2^{n / 2}=2^{n}$.

To summarize, when $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right)$ is satisfied then $\operatorname{DDT}_{F}[\alpha, \beta]$ can be computed as follows:
$\operatorname{DDT}_{F}[\alpha, \beta]= \begin{cases}2^{n}, & \text { if } \alpha \in\left\{\mathbf{0},\left(\mathbf{0}, k_{R}\right),\left(k_{L}, k_{L}\right),\left(k_{L}, k_{L} \oplus k_{R}\right)\right\}, \\ 2^{n-1}, & \text { if }\left(\alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \in\left\{\mathbf{0}, k_{R}\right\}\right) \text { or }\left(\alpha_{L} \in\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\}\right), \\ 2^{n-2}, \text { if } \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\} .\end{cases}$
The same method can be used to address the remaining cases, and the following results are then obtained.

Case 2. $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus\left(0, k_{L}\right)$.

$$
\operatorname{DDT}_{F}[\alpha, \beta]= \begin{cases}2^{n-1}, & \text { if } \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \in\left\{\mathbf{0}, k_{R}\right\},  \tag{5}\\ 2^{n-2}, & \text { if } \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\}, \\ 0, & \text { if } \alpha_{L} \in\left\{\mathbf{0}, k_{L}\right\} .\end{cases}
$$

Case 3. $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus\left(k_{R}, 0\right)$

$$
\operatorname{DDT}_{F}[\alpha, \beta]= \begin{cases}2^{n-1}, & \text { if } \alpha_{L} \in\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\},  \tag{6}\\ 2^{n-2}, & \text { if } \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\}, \\ 0, & \text { if } \alpha_{L} \oplus \alpha_{R} \in\left\{\mathbf{0}, k_{R}\right\} .\end{cases}
$$

Case 4. $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus\left(k_{R}, k_{L}\right)$

$$
\mathrm{DDT}_{F}[\alpha, \beta]= \begin{cases}2^{n-2}, & \text { if } \alpha_{L} \notin\left\{\mathbf{0}, k_{L}\right\} \text { and } \alpha_{L} \oplus \alpha_{R} \notin\left\{\mathbf{0}, k_{R}\right\},  \tag{7}\\ 0, & \text { if } \alpha_{L} \in\left\{\mathbf{0}, k_{L}\right\} \text { or } \alpha_{L} \oplus \alpha_{R} \in\left\{\mathbf{0}, k_{R}\right\} .\end{cases}
$$

By (4), (5), (6), and (7), the DDT of $F(x)$ can be obtained.\#
Moreover, we consider the differential properties when the round function is applied iteratively. It is well-known that the probability of a differential characteristic of Markov cipher [20] can be easily calculated. In what follows, we first prove that DBISON is a Markov cipher.

Lemma 1 The round function $F_{k, w}(x)$ of DBISON has the following property

$$
\begin{equation*}
\operatorname{Pr}_{w}\left[F_{k, w}(x) \oplus F_{k, w}(x \oplus \alpha)=\beta\right]=\operatorname{Pr}_{x}\left[F_{k, w}(x) \oplus F_{k, w}(x \oplus \alpha)=\beta\right] . \tag{8}
\end{equation*}
$$

Proof Let $A_{w}:=\left\{w \in F_{2}^{n-2} \mid F_{k, w}(x) \oplus F_{k, w}(x \oplus \alpha)=\beta\right\}, A_{x}:=\left\{x \in F_{2}^{n} \mid F_{k, w}(x) \oplus F_{k, w}(x \oplus \alpha)=\beta\right\}$.

## More specifically,

$$
\begin{aligned}
A_{w}= & \left\{w \in F_{2}^{n-2} \mid\left(D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right) k_{R}, D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right)=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta\right\} . \\
A_{x}= & \left\{x \in F_{2}^{n} \mid\left(D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right) k_{R}, D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right)=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta\right\} . \\
& \text { If }\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta \notin K^{*} \text {, then }\left|A_{w}\right|=\left|A_{\chi}\right|=0 \text {, and (8) holds. If }\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right) \oplus \beta \in K^{*},
\end{aligned}
$$ then $\left|A_{w}\right|$ and $\left|A_{\lambda}\right|$ are calculated as below.

Case 1. $\beta=\left(\alpha_{L} \oplus \alpha_{R}, \alpha_{L}\right)$.

$$
A_{w}=\left\{w_{L} \in F_{2}^{n / 2-1} \mid D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=0\right\} \times\left\{w_{R} \in F_{2}^{n / 2-1} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right)=0\right\}
$$

Denote $w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)=u$ and $w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)=v$, then

$$
\begin{aligned}
A_{w} & =\left\{u \oplus \Phi_{k_{L}}\left(x_{L}\right) \in F_{2}^{n / 2-1} \mid D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}(u)=0\right\} \times\left\{v \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right) \in F_{2}^{n / 2-1} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}(v)=0\right\} \\
& =\left[\Phi_{k_{L}}\left(x_{L}\right) \oplus\left(F_{2}^{n / 2-1}-\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right)\right] \times\left[\Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right) \oplus\left(F_{2}^{n / 2-1}-\operatorname{supp}\left(D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\right)\right)\right] .
\end{aligned}
$$

Thus,

$$
\operatorname{Pr}_{w}\left[F_{k, w}(x) \oplus F_{k, w}(x \oplus \alpha)=\beta\right]=\frac{\left|A_{w}\right|}{\left|F_{2}^{n-2}\right|}=\frac{\left(2^{n / 2-1}-\left|\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right|\right)\left(2^{n / 2-1}-\left|\operatorname{supp}\left(D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\right)\right|\right)}{2^{n-2}}
$$

On the other hand, $\left|A_{x}\right|$ can be calculated as follows.

$$
A_{x}=\left\{\left(x_{L}, x_{R}\right) \in F_{2}^{n} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right)=0 \text { and } D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=0\right\} .
$$

If $\Phi_{k_{L}}\left(\alpha_{L}\right)=\mathbf{0}$, then $\left|\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right|=0$ and $\left|A_{L}\right|=2^{n / 2}$. If $\Phi_{k_{L}}\left(\alpha_{L}\right) \neq \mathbf{0}$, it can be deduced that $\left|\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right|=2^{n / 2-2}$ since $f_{L}$ is a bent function, and $\left|A_{L}\right|=2^{n / 2-1}$ (see Theorem 1). In both cases, $\left|A_{L}\right|=2^{n / 2}-2\left|\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right|$.

For any $a_{i} \in A_{L}, i=1,2, \ldots, 2^{n / 2-1}$, if $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)=\mathbf{0}$, then $\left|\operatorname{supp}\left(D_{\Phi_{{k_{R}}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\right)\right|=0$ and $\left|\left\{x_{R} \in F_{2}^{n / 2} \mid D_{\Phi_{k_{R}\left(\alpha_{L} \oplus \alpha_{R}\right)}} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(a_{i} \oplus x_{R}\right)\right)=0\right\}\right|=2^{n / 2}$. If $\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right) \neq \mathbf{0}$, it can be deduced that $\left|\operatorname{supp}\left(D_{\left.\Phi_{k_{R}\left(\alpha_{L} \oplus \alpha_{R}\right)}\right)} f_{R}\right)\right|=2^{n / 2-2}$, and $\left|\left\{x_{R} \in F_{2}^{n / 2} \mid D_{\Phi_{k_{R}( }\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(a_{i} \oplus x_{R}\right)\right)=0\right\}\right|=2^{n / 2-1}$. In both cases, $\left|\left\{x_{R} \in F_{2}^{n / 2} \mid D_{\Phi_{k_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(a_{i} \oplus x_{R}\right)\right)=0\right\}\right|=2^{n / 2}-2\left|\operatorname{supp}\left(D_{\Phi_{\kappa_{R}}\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\right)\right|$.

To summarize, $\left|A_{\chi}\right|=\left(2^{n / 2}-2\left|\operatorname{supp}\left(D_{\Phi_{k_{L}}\left(\alpha_{L}\right)} f_{L}\right)\right|\right)\left(2^{n / 2}-2\left|\operatorname{supp}\left(D_{\Phi_{K_{R}( }\left(\alpha_{L} \oplus \alpha_{R}\right)} f_{R}\right)\right|\right)$, thus (8) holds. The similar results are easily verified for the remaining cases.\#

Corollary 1 Let $E_{k, w}^{r}$ denote the $r$-round encryption of DBISON, where its $i$-th round function is $F_{k_{i}, w_{i}}(x)$ and using the round keys $k_{1}, k_{2}, \ldots, k_{r}$. Then, we have

$$
\operatorname{Pr}_{w, x}\left[E_{k, w}^{r}(x) \oplus E_{k, w}^{r}\left(x \oplus \delta_{0}\right)=\delta_{r}\right]=\prod_{i=1}^{r} \operatorname{Pr}_{w, x}\left[F_{k_{i}, w_{i}}(x) \oplus F_{k_{i}, w_{i}}\left(x \oplus \delta_{i-1}\right)=\delta_{i}\right] .
$$

To describe the necessary conditions under which $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right)$ is a valid differential characteristic and to compute the MDP of DBISON, we need to introduce a new operation.

Definition 5 Let $\lambda_{L}, \lambda_{R} \in\{0,1\},\left(k_{L}, k_{R}\right) \in F_{2}^{n}, k_{L} \in F_{2}^{n / 2}, k_{R} \in F_{2}^{n / 2}$. We define a "product", between $\left(\lambda_{L}, \lambda_{R}\right)$ and $\left(k_{L}, k_{R}\right)$ as $\left(\lambda_{L}, \lambda_{R}\right) *\left(k_{L}, k_{R}\right)=\left(\lambda_{L} k_{L}, \lambda_{R} k_{R}\right)$.

By Corollary 1, the probability of having the differential characteristic $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right)$ after $r$ rounds is $\operatorname{DP}(\Omega)=\prod_{i=1}^{r} \mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right]$. In particular, $\operatorname{DP}(\Omega)=0$ if and only if there is $0 \leq j \leq r$, such that $\operatorname{DP}\left[\delta_{j-1}, \delta_{j}\right]=0$. By Theorem 1, $\operatorname{DDT}_{F}\left[\delta_{i-1}, \delta_{i}\right]=0$ if

$$
\delta_{i} \notin\left\{\left(\delta_{(i-1) L}+\delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus \gamma \mid \gamma \in\left\{\mathbf{0},\left(\mathbf{0}, k_{i L}\right),\left(k_{i R}, \mathbf{0}\right),\left(k_{i R}, k_{i L}\right)\right\}\right\},
$$

which means $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=0$. Moreover, a valid differential characteristic $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right)$ should have the following form.

$$
\begin{equation*}
\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{r}\right), \delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(\lambda_{i L}, \lambda_{i R}\right) *\left(k_{i R}, k_{i L}\right), \tag{9}
\end{equation*}
$$

where $\lambda_{i L}, \lambda_{i R} \in\{0,1\}$, and $k_{i}=\left(k_{i L}, k_{i R}\right)$ is the round key.
Theorem 2 For n-round DBISON, if the round keys satisfy $k_{i R} \notin\left\{k_{(i-1) L}, k_{(i+1) L}\right\}$, then there is no nontrivial differential characteristic whose probability equals 1.
Proof Assume $\Omega=\left(\delta_{0}, \delta_{1}, \ldots, \delta_{n}\right)$ is a nontrivial differential characteristic in (9) and $D P[\Omega]=1$, thus $\operatorname{DP}\left(\delta_{i-1}, \delta_{i}\right)=1, i=1,2, \ldots, n$. Especially, $\operatorname{DP}\left(\delta_{0}, \delta_{1}\right)=\mathrm{DP}\left(\delta_{1}, \delta_{2}\right)=1$. By Theorem 1, $\mathrm{DP}\left[\delta_{0}, \delta_{1}\right]=1$ if and only if $\delta_{1}=\left(\delta_{0 L} \oplus \delta_{0 R}, \delta_{0 L}\right)$ and $\delta_{0} \in\left\{\mathbf{0},\left(\mathbf{0}, k_{1 R}\right),\left(k_{1 L}, k_{1 L}\right),\left(k_{1 L}, k_{1 L} \oplus k_{1 R}\right)\right\}$.

If $\delta_{0}=0$, by Theorem 1 , it can be deduced that $\delta_{1}=\delta_{2}=\ldots=\delta_{n}=0$, thus $\Omega$ is a trivial differential characteristic that holds with probability 1, which contradicts the assumption.

If $\delta_{0}=\left(\mathbf{0}, k_{1 R}\right)$, then $\delta_{1}=\left(\mathbf{0} \oplus k_{1 R}, \mathbf{0}\right)$. Using $\operatorname{DP}\left(\delta_{1}, \delta_{2}\right)=1$ and Theorem 1, we have

$$
\left(k_{1 R}, \mathbf{0}\right)=\delta_{1} \in\left\{\mathbf{0},\left(\mathbf{0}, k_{2 R}\right),\left(k_{2 L}, k_{2 L}\right),\left(k_{2 L}, k_{2 L} \oplus k_{2 R}\right)\right\} .
$$

This contradicts the conditions that $k_{1 R} \neq \mathbf{0}$ and $k_{1 R} \neq k_{2 L}$.
If $\delta_{0}=\left(k_{1 L}, k_{1 L}\right)$, then $\delta_{1}=\left(k_{1 L} \oplus k_{1 L}, k_{1 L}\right)=\left(\mathbf{0}, k_{1 L}\right) . \operatorname{From~DP}\left(\delta_{1}, \delta_{2}\right)=1$ and Theorem 1, it can be deduced that

$$
\left(\mathbf{0}, k_{1 L}\right)=\delta_{1} \in\left\{\mathbf{0},\left(\mathbf{0}, k_{2 R}\right),\left(k_{2 L}, k_{2 L}\right),\left(k_{2 L}, k_{2 L} \oplus k_{2 R}\right)\right\} .
$$

This contradicts the conditions that $k_{i L} \neq \mathbf{0}$ and $k_{2 R} \neq k_{1 L}$.
If $\delta_{0}=\left(k_{1 L}, k_{1 L} \oplus k_{1 R}\right)$, then $\delta_{1}=\left(k_{1 L} \oplus k_{1 R} \oplus k_{1 L}, k_{1 L}\right)=\left(k_{1 R}, k_{1 L}\right)$. Using $\operatorname{DP}\left(\delta_{1}, \delta_{2}\right)=1$ and Theorem 1, it can be deduced that

$$
\left(k_{1 R}, k_{1 L}\right)=\delta_{1} \in\left\{\mathbf{0},\left(\mathbf{0}, k_{2 R}\right),\left(k_{2 L}, k_{2 L}\right),\left(k_{2 L}, k_{2 L} \oplus k_{2 R}\right)\right\} .
$$

Again, this violates the conditions that $k_{1 R} \neq \mathbf{0}$ and $k_{1 R} \neq k_{2 L}$.
From the above cases, it can be concluded that there is no nontrivial differential characteristic with probability 1.\#

To prove that DBISON is resistant against differential cryptanalysis, we need to analyze its MDP.

Theorem 3 For the differential characteristic $\Omega$ given by (9), we have:

1) If there is $\delta_{j}=\mathbf{0}$ and $\delta_{j+1} \neq \mathbf{0}$, then $\mathrm{DP}[\Omega]=0$.
2) If there is $\delta_{j}=\mathbf{0}$ and $\delta_{j-1} \neq \mathbf{0}$, then $\mathrm{DP}[\Omega]=0$.

## Proof

1) By (9), using $\delta_{j}=\mathbf{0}$ and $\delta_{j+1} \neq \mathbf{0}$, it can be deduced that

$$
\delta_{j+1}=\left(\lambda_{j L}, \lambda_{j R}\right) *\left(k_{j R}, k_{j L}\right) \in\left\{\left(k_{j R}, \mathbf{0}\right),\left(\mathbf{0}, k_{j L}\right),\left(k_{j R}, k_{j L}\right)\right\} .
$$

If $\delta_{j+1}=\left(k_{j R}, \mathbf{0}\right)$, then $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n} \quad$ since $\quad \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right)$. Also, $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-2}$, since we can represent $\delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(k_{j R}, \mathbf{0}\right)$ and the assumption $\delta_{j L}=\mathbf{0}$ contradicts Thereom 1. Moreover, $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-1}$, since $\delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right), \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(\mathbf{0}, k_{j L}\right)$, and representing $\delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus$ $\left(k_{j R}, \mathbf{0}\right)$ along with $\delta_{j L} \oplus \delta_{j R}=\mathbf{0}$ implies that $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-1}$.

If $\delta_{j+1}=\left(\mathbf{0}, k_{j L}\right)$, then $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n} \quad$ since $\delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right)$. Similarly,
$\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-2}$ since $\delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(\mathbf{0}, k_{j L}\right)$ and $\delta_{j L}=\mathbf{0}$. Also, $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-1}$ since $\delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right), \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(k_{j R}, \mathbf{0}\right)$ and expressing $\delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus$ $\left(\mathbf{0}, k_{j L}\right)$ along with the assumption $\delta_{j L}=\mathbf{0}$ proves the claim.

If $\delta_{j+1}=\left(k_{j R}, k_{j L}\right)$, then $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n} \quad$ since $\quad \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right)$. Also, $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-2} \quad$ since $\quad \delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(k_{j R}, k_{j L}\right) \quad$ but $\quad \delta_{j L}=\mathbf{0} \quad$. Finally, $\operatorname{DDT}\left[\delta_{j}, \delta_{j+1}\right] \neq 2^{n-1} \quad$ since $\quad \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \quad, \quad \delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(\mathbf{0}, k_{j L}\right) \quad$ and $\delta_{j+1} \neq\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right) \oplus\left(k_{j R}, \mathbf{0}\right)$.

Therefore, $\mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right]=0$, and moreover $\mathrm{DP}[\Omega]=0$.
The proof of 2 ) is similar to the proof of 1 ). \#
Actually, from the result of Theorem 3 , we only need to consider $\Omega$ in (9) when $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$.

Theorem 4 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristics given by (9) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Let also the round keys satisfy $k_{i R} \notin\left\{k_{(i-1) L}, k_{i L}, k_{(i+1) L}, k_{i L} \oplus k_{(i-1) L}\right\}$. If there is $\delta_{j-1}$ such that $\mathrm{DP}\left[\delta_{j-1}, \delta_{j}\right]=1$, then $\mathrm{DP}\left[\delta_{j-2}, \delta_{j-1}\right] \neq 1$ and $\mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right] \neq 1$.
Proof By Theorem 1 and using $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$, it is clear that $\mathrm{DP}\left[\delta_{j-1}, \delta_{j}\right]=1$ if and only if $\delta_{j}=\left(\delta_{(j-1) L} \oplus \delta_{(j-1) R}, \delta_{(j-1) L}\right)$ and $\delta_{j-1} \in\left\{\left(\mathbf{0}, k_{j R}\right),\left(k_{j L}, k_{j L}\right),\left(k_{j L}, k_{j L} \oplus k_{j R}\right)\right\} . \mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right] \neq 1$ can be proved using reduction to the absurd, the proof of $\mathrm{DP}\left[\delta_{j-2}, \delta_{j-1}\right] \neq 1$ is similar, thus it is omitted here.

Now, assuming that $\mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$, by Theorem $1, \mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$ if and only if $\delta_{j+1}=\left(\delta_{j L} \oplus \delta_{j R}, \delta_{j L}\right)$ and $\delta_{j} \in A_{\delta_{j}}:=\left\{\left(\mathbf{0}, k_{(j+1) R}\right),\left(k_{(j+1) L}, k_{(j+1) L}\right),\left(k_{(j+1) L}, k_{(j+1) L} \oplus k_{(j+1) R}\right)\right\}$.

If $\delta_{j-1}=\left(\mathbf{0}, k_{j R}\right)$, using that $\mathrm{DP}\left[\delta_{j-1}, \delta_{j}\right]=1$, we get $\delta_{j}=\left(\mathbf{0} \oplus k_{j R}, \mathbf{0}\right)=\left(k_{j R}, \mathbf{0}\right)$. Combining this with $\operatorname{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$, we have $\left(k_{j R}, \mathbf{0}\right)=\delta_{j} \in A_{\delta_{j}}$ which contradicts the condition that $k_{i R} \neq k_{(i+1) L}$.

If $\delta_{j-1}=\left(k_{j L}, k_{j L}\right)$, using that $\operatorname{DP}\left[\delta_{j-1}, \delta_{j}\right]=1$, we get $\delta_{j}=\left(k_{j L} \oplus k_{j L}, k_{j L}\right)=\left(\mathbf{0}, k_{j L}\right)$. Combining this with $\operatorname{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$, we have $\left(\mathbf{0}, k_{j L}\right)=\delta_{j} \in A_{\delta_{j}}$ which contradicts the condition that $k_{i R} \neq k_{(i-1) L}$.

If $\delta_{j-1}=\left(k_{j L}, k_{j L} \oplus k_{j R}\right)$, using that $\operatorname{DP}\left[\delta_{j-1}, \delta_{j}\right]=1$, we get

$$
\delta_{j}=\left(k_{j L} \oplus k_{j R} \oplus k_{j L}, k_{j L}\right)=\left(k_{j R}, k_{j L}\right) .
$$

Again, combining this with $\operatorname{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$, we have $\left(k_{j R}, k_{j L}\right)=\delta_{j} \in A_{\delta_{j}}$ which contradicts the condition that $k_{i R} \neq k_{(i+1) L}$.

Therefore, the assumption that $\mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right]=1$ does not hold, thus $\mathrm{DP}\left[\delta_{j}, \delta_{j+1}\right] \neq 1$. \#
By Theorem 4, we know that any two consecutive factors of $\mathrm{DP}[\Omega]=\prod_{i=1}^{n} \mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right]$
cannot be 1 simultaneously, hence there are at most $n / 2$ multiplicative factors that are equal 1. Moreover, because $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right] \in\left\{0,1 / 2^{2}, 1 / 2,1\right\}$, it is clear that $\mathrm{DP}[\Omega] \leq 2^{-n / 2}$.

Theorem 5 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristic given by (9), with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Let the round keys satisfy:

$$
k_{i R} \notin\left\{k_{(i-1) L}, k_{i L}, k_{(i+1) L}, k_{(i-1) L} \oplus k_{i L}, k_{(i-2) R}\right\} \text { and } k_{i L} \neq k_{(i+1) L} .
$$

If $\mathrm{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\mathrm{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, then $\mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right] \neq 1 / 2$.
Proof Assume $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2$. By Theorem 1, $\mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2$ if and only if one of the following cases occurs.

## Case 1.

$$
\begin{aligned}
\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \in & \left\{\left(\mathbf{0}, \delta_{(i-1) L}\right),\left(k_{i R}, \delta_{(i-1) L}\right),\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \mathbf{0}\right),\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, k_{i L}\right)\right. \\
& \left.\mid \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}, \delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}\right\}:=A_{1} .
\end{aligned}
$$

Using $\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we get $\delta_{i} \in A_{\delta_{i}}:=\left\{\left(\mathbf{0}, k_{(i+1) R}\right),\left(k_{(i+1) L}, k_{(i+1) L}\right),\left(k_{(i+1) L}, k_{(i+1) L} \oplus k_{(i+1) R}\right)\right\}$. Due to the conditions that the round keys satisfy, $A_{1} \cap A_{\delta_{i}} \neq \varnothing$ if and only if $\delta_{(i-1) L}=k_{(i+1) R}$. However, using $\operatorname{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=1$, we get $\delta_{(i-1) L}=\delta_{(i-2) L} \oplus \delta_{(i-2) R} \in\left\{k_{(i-1) R}, 0\right\}$ which means $k_{(i+1) R}=k_{(i-1) R}$, a contradiction.

Case 2. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(\mathbf{0}, k_{i L}\right), \delta_{(i-1) L} \oplus \delta_{(i-1) R} \in\left\{\mathbf{0}, k_{i R}\right\}$ and $\delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}$.
In this case, $\quad \delta_{i} \in\left\{\left(\mathbf{0}, \delta_{(i-1) L} \oplus k_{i L}\right),\left(k_{i R}, \delta_{(i-1) L} \oplus k_{i L}\right) \mid \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}\right\}:=A_{2} \quad$ Using $\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we get $\delta_{i} \in A_{\delta_{i}} . A_{1} \cap A_{\delta_{i}} \neq \varnothing$ if and only if $\delta_{(i-1) L} \oplus k_{i L}=k_{(i+1) R}$. However, since $\operatorname{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=1$, then $\delta_{(i-1) L}=\delta_{(i-2) L} \oplus \delta_{(i-2) R} \in\left\{k_{(i-1) R}, \mathbf{0}\right\}$ which implies that $k_{i L}=k_{(i+1) R} \oplus k_{(i-1) R}$ or $k_{(i+1) R}$, a contradiction.

Case 3. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(k_{i R}, \mathbf{0}\right), \delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}$ and $\delta_{(i-1) L} \in\left\{\mathbf{0}, k_{i L}\right\}$.
In this case, $\quad \delta_{i} \in\left\{\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R} \oplus k_{i R}, \mathbf{0}\right),\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R} \oplus k_{i R}, k_{i L}\right) \mid \delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}\right\}$ $:=A_{3}$. By $\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we have $\delta_{i} \in A_{\delta_{i}}$. Then, the conditions imposed on the round keys imply that $A_{3} \cap A_{\delta_{i}}=\varnothing$.

To summarize, the assumption $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2$ cannot hold. \#
Remark 2 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristic given by (9) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Assuming that the round keys satisfy the conditions of Theorem 5 , we cannot possibly have the case $D P[\Omega]=1 \times(1 / 2) \times 1 \times(1 / 2) \times 1 \ldots$.

Theorem 6 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristic given by (9) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Assume that the round keys satisfy

1) $\quad k_{i R} \notin\left\{k_{(i-1) L}, k_{i L}, k_{(i+2) L}, k_{(i-2) L} \oplus k_{(i-1) L}, k_{(i-1) L} \oplus k_{i L}, k_{i L} \oplus k_{(i+2) L}, k_{(i-1) R}\right\}$.
2) $k_{i L} \notin\left\{k_{(i-2) L}, k_{(i-1) L}, k_{i R} \oplus k_{(i+1) R}, k_{(i+1) R} \oplus k_{(i+2) R}\right\}$.
3) $\quad k_{(i-1) L} \oplus k_{i L} \neq k_{i R} \oplus k_{(i+1) R}$.

If $\operatorname{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\mathrm{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, then $\mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right] \neq 1 / 2^{2}$.

Proof Assume $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2^{2}$. By Theorem 1, $\mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2^{2}$ if and only if one of the following cases occurs.

Case 1. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right), \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}$ and $\delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}$
Using $\operatorname{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, one can deduce:

$$
\delta_{i+1}=\left(\delta_{i L} \oplus \delta_{i R}, \delta_{i L}\right)=\left(\delta_{(i-1) R}, \delta_{(i-1) L} \oplus \delta_{(i-1) R}\right)=\left(\delta_{(i-2) L}, \delta_{(i-2) R}\right),
$$

where $\delta_{i L} \in\left\{\mathbf{0}, k_{(i+1) L}\right\}:=B_{1}$, and $\delta_{(i-2) R} \in\left\{k_{(i-1) R}, k_{(i-1) L}, k_{(i-1) L} \oplus k_{(i-1) R}\right\}:=B_{2}$. The conditions on the round keys imply that $B_{1} \cap B_{2}=\varnothing$, which contradicts the fact that $\delta_{i L}=\delta_{(i-2) R}$.

Case 2. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(\mathbf{0}, k_{i L}\right), \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}$ and $\delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}$
Using $\operatorname{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we get the following equation

$$
\delta_{i+1}=\left(\delta_{i L} \oplus \delta_{i R}, \delta_{i L}\right)=\left(\delta_{(i-1) R} \oplus k_{i L}, \delta_{(i-1) L} \oplus \delta_{(i-1) R}\right)=\left(\delta_{(i-2) L} \oplus k_{i L}, \delta_{(i-2) R}\right) \text {, }
$$

where $\delta_{i L} \oplus \delta_{i R} \in\left\{\mathbf{0}, k_{(i+1) R}\right\}:=B_{3}$, and $\delta_{(i-2) L} \oplus k_{i L} \in\left\{k_{i L}, k_{(i-1) L} \oplus k_{i L}\right\}:=B_{4}$. The conditions on the round keys give that $B_{3} \cap B_{4}=\varnothing$, which contradicts the fact that $\delta_{i L} \oplus \delta_{i R}=\delta_{(i-2) L} \oplus k_{i L}$.

Case 3. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(k_{i R}, \mathbf{0}\right), \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}$ and $\delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}$
Using $\mathrm{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\mathrm{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we have

$$
\delta_{i+1}=\left(\delta_{i L} \oplus \delta_{i R}, \delta_{i L}\right)=\left(\delta_{(i-1) R} \oplus k_{i R}, \delta_{(i-1) L} \oplus \delta_{(i-1) R} \oplus k_{i R}\right)=\left(\delta_{(i-2) L} \oplus k_{i R}, \delta_{(i-2) R} \oplus k_{i R}\right),
$$

where $\delta_{i L} \oplus \delta_{i R} \in B_{3}$, and $\delta_{(i-2) L} \oplus k_{i R} \in\left\{k_{i R}, k_{(i-1) L} \oplus k_{i R}\right\}:=B_{5}$. The assumptions on the round keys give that $B_{3} \cap B_{5}=\varnothing$, which contradicts $\delta_{i L} \oplus \delta_{i R}=\delta_{(i-2) L} \oplus k_{i R}$.

Case 4. $\delta_{i}=\left(\delta_{(i-1) L} \oplus \delta_{(i-1) R}, \delta_{(i-1) L}\right) \oplus\left(k_{i R}, k_{i L}\right), \delta_{(i-1) L} \notin\left\{\mathbf{0}, k_{i L}\right\}$ and $\delta_{(i-1) L} \oplus \delta_{(i-1) R} \notin\left\{\mathbf{0}, k_{i R}\right\}$
Again, using $\mathrm{DP}\left[\delta_{i-2}, \delta_{i-1}\right]=\mathrm{DP}\left[\delta_{i}, \delta_{i+1}\right]=1$, we obtain

$$
\delta_{i+1}=\left(\delta_{i L} \oplus \delta_{i R}, \delta_{i L}\right)=\left(\delta_{(i-1) R} \oplus k_{i R} \oplus k_{i L}, \delta_{(i-1) L} \oplus \delta_{(i-1) R} \oplus k_{i R}\right)=\left(\delta_{(i-2) L} \oplus k_{i R} \oplus k_{i L}, \delta_{(i-2) R} \oplus k_{i R}\right),
$$

where $\delta_{i L} \oplus \delta_{i R} \in B_{3}, \delta_{(i-2) L} \oplus k_{i R} \oplus k_{i L} \in\left\{k_{i R} \oplus k_{i L}, k_{i R} \oplus k_{(i-1) L} \oplus k_{i L}\right\}:=B_{6}$. Similarly as above, we get $B_{3} \cap B_{6}=\varnothing$ which contradicts $\delta_{i L} \oplus \delta_{i R}=\delta_{(i-2) L} \oplus k_{i R} \oplus k_{i L}$.

Therefore, the assumption that $\mathrm{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 / 2^{2}$ cannot hold. \#
Remark 3 For $n$-round DBISON, let $\Omega$ denote the $n$-round differential characteristic given by (9) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Assuming that the round keys satisfy conditions in Theorem 6, it is impossible to have $D P[\Omega]=1 \times\left(1 / 2^{2}\right) \times 1 \times\left(1 / 2^{2}\right) \times 1 \ldots$.

Theorem 7 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristic given by (9), with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Assume that the round keys satisfy $k_{i R} \notin\left\{k_{(i+1) R}, k_{i L}, k_{(i+1) L}\right\}$ and $k_{i L} \notin\left\{k_{(i+1) R}, k_{i R} \oplus k_{(i+1) R}\right\}$. If $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=\mathrm{DP}\left[\delta_{i+2}, \delta_{i+3}\right]=1$, then the following equalities cannot hold: $\mathrm{DP}\left[\delta_{i}, \delta_{i+1}\right]=\mathrm{DP}\left[\delta_{i+1}, \delta_{i+2}\right]=1 / 2$.
Proof By Theorem 1, the conditions on the round keys, and $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1$, one can deduce that $\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=1 / 2$ if and only if $\delta_{i-1}=\left(k_{i L}, k_{i L}\right), \delta_{i}=\left(\mathbf{0}, k_{i L}\right)$, and $\delta_{i+1}=\left(k_{i L} \oplus k_{(i+1) R}, \mathbf{0}\right)$. Furthermore, $\operatorname{DP}\left[\delta_{i+1}, \delta_{i+2}\right]=1 / 2$ holds if and only if $\delta_{i+2}=\left(k_{i L} \oplus k_{(i+1) R} \oplus k_{(i+2) R}, k_{i L} \oplus k_{(i+1) R}\right)$ or
$\delta_{i+2}=\left(k_{i L} \oplus k_{(i+1) R}, k_{i L} \oplus k_{(i+2) L} \oplus k_{(i+1) R}\right)$.
If $\delta_{i+2}=\left(k_{i L} \oplus k_{(i+1) R} \oplus k_{(i+2) R}, k_{i L} \oplus k_{(i+1) R}\right)$, then from Theorem 1 and $\operatorname{DP}\left[\delta_{i+2}, \delta_{i+3}\right]=1$, it can be easily verified that

$$
\delta_{i+3}=\left(\delta_{(i+2) L} \oplus \delta_{(i+2) R}, \delta_{(i+2) L}\right) \text { and } \delta_{i+2} \in\left\{\left(\mathbf{0}, k_{(i+3) R}\right),\left(k_{(i+3) L}, k_{(i+3) L}\right),\left(k_{(i+3) L}, k_{(i+3) R} \oplus k_{(i+3) L}\right)\right\} \text {. }
$$

This means that

$$
\left(k_{i L} \oplus k_{(i+1) R} \oplus k_{(i+2) R}, k_{i L} \oplus k_{(i+1) R}\right) \in\left\{\left(0, k_{(i+3) R}\right),\left(k_{(i+3) L}, k_{(i+3) L}\right),\left(k_{(i+3) L}, k_{(i+3) R} \oplus k_{(i+3) L}\right)\right\},
$$

which contradicts the assumptions on the round keys. If $\delta_{i+2}=\left(k_{i L} \oplus k_{(i+1) R}, k_{i L} \oplus k_{(i+2) L} \oplus k_{(i+1) R}\right)$, a similar conclusion is valid. \#

Generalizing the conclusions given in Theorem 7, we observe the following.
Remark 4 For $n$-round DBISON, let $\Omega$ be the $n$-round differential characteristic given by (9) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$. Assuming that the round keys $k_{i}=\left(k_{i L}, k_{i R}\right)$ satisfy the conditions that $k_{i R}$ is linearly independent from $k_{i L}, k_{(i+1) L}, \ldots, k_{(i+l-2) L}$ and $k_{i L}$ is linearly independent from $k_{i R}, k_{(i+1) R}, \ldots, k_{(i+1-2) R}$, then $\operatorname{DP}\left[\delta_{i}, \delta_{i+1}\right]=\mathrm{DP}\left[\delta_{i+1}, \delta_{i+2}\right]=\ldots=\operatorname{DP}\left[\delta_{i+1-2}, \delta_{i+l-1}\right]=\frac{1}{2}$ and $\operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=\operatorname{DP}\left[\delta_{i+1-1}, \delta_{i+1}\right]=1$ cannot hold.

By Remarks 2, 3, 4, for $n$-round DBISON (whose round keys satisfy certain conditions) and $\Omega$ described by ( 9 ) with $\delta_{i} \neq \mathbf{0}, i=1,2, \ldots, n$, if there exists a differential characteristic of the form

$$
\mathrm{DP}[\Omega]=\prod_{i=1}^{n} \operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 \times(1 / 2) \times\left(1 / 2^{2}\right) \times 1 \times(1 / 2) \times\left(1 / 2^{2}\right) \times 1 \ldots,
$$

then the probability of this differential characteristic is maximal. Then,

$$
\prod_{i=1}^{n} \operatorname{DP}\left[\delta_{i-1}, \delta_{i}\right]=1 \times(1 / 2) \times\left(1 / 2^{2}\right) \times 1 \times(1 / 2) \times\left(1 / 2^{2}\right) \times 1 \ldots=1^{[n / 3]}(1 / 2)^{[(n-1) / 3]}\left(1 / 2^{2}\right)^{[n-2) / 3]}:=h(n) .
$$

Table 2 gives some values of $h(n)$ for different $n$. Most notably, $h(n)=1 / 2^{n}$ if $n$ is divisible by 6 , otherwise, $h(n)=1 / 2^{n-1}$.

Remark 5 For $n$-round DBISON, we have MDP $\leq 1 / 2^{n-1}$ when the round keys satisfy the conditions given in the previous theorems. Therefore, we conclude that $n$-round DBISON is resistant against differential cryptanalysis.

Table 2. Values of h(n)

| $n$ | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 58 | 62 | 66 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~h}(n)$ | $2^{-6}$ | $2^{-9}$ | $2^{-13}$ | $2^{-18}$ | $2^{-21}$ | $2^{-25}$ | $2^{-30}$ | $2^{-33}$ | $2^{-37}$ | $2^{-42}$ | $2^{-45}$ | $2^{-49}$ | $2^{-54}$ | $2^{-57}$ | $2^{-61}$ | $2^{-66}$ |
| $n$ | 70 | 74 | 78 | 82 | 86 | 90 | 94 | 98 | 102 | 106 | 110 | 114 | 118 | 122 | 126 | 130 |
| $\mathrm{~h}(n)$ | $2^{-69}$ | $2^{-73}$ | $2^{-78}$ | $2^{-81}$ | $2^{-85}$ | $2^{-90}$ | $2^{-93}$ | $2^{-97}$ | $2^{-102}$ | $2^{-105}$ | $2^{-109}$ | $2^{-114}$ | $2^{-117}$ | $2^{-121}$ | $2^{-126}$ | $2^{-129}$ |
| $n$ | 134 | 138 | 142 | 146 | 150 | 154 | 158 | 162 | 166 | 170 | 174 | 178 | 182 | 186 | 190 | 194 |
| $\mathrm{~h}(n)$ | $2^{-133}$ | $2^{-138}$ | $2^{-141}$ | $2^{-145}$ | $2^{-150}$ | $2^{-153}$ | $2^{-157}$ | $2^{-162}$ | $2^{-165}$ | $2^{-169}$ | $2^{-174}$ | $2^{-177}$ | $2^{-181}$ | $2^{-186}$ | $2^{-189}$ | $2^{-193}$ |
| $n$ | 198 | 202 | 206 | 210 | 214 | 218 | 222 | 226 | 230 | 234 | 238 | 242 | 246 | 250 | 254 | 258 |
| $\mathrm{~h}(n)$ | $2^{-198}$ | $2^{-201}$ | $2^{-205}$ | $2^{-210}$ | $2^{-213}$ | $2^{-217}$ | $2^{-222}$ | $2^{-225}$ | $2^{-229}$ | $2^{-234}$ | $2^{-237}$ | $2^{-241}$ | $2^{-246}$ | $2^{-249}$ | $2^{-253}$ | $2^{-258}$ |

## 4. Linear cryptanalysis of the DBISON block cipher

To evaluate the resistance of DBISON against linear cryptanalysis, we need to specify the linear approximation table (LAT) of the round function $F_{k, w}(x)$. Recall that $F_{k, w}(x)$ was defined in (1), where the linear functions $\Phi_{k_{i L}}$ and $\Phi_{k_{i R}}$ are given by:

$$
\begin{align*}
& \Phi_{k_{L}}\left(x_{L}\right)=\left(x_{L_{\left(k_{L}\right)}} k_{L} \oplus x_{L}\right)\left[1, \ldots, i\left(k_{L}\right)-1, i\left(k_{L}\right)+1, \ldots, n / 2\right] \\
& \Phi_{k_{R}}\left(x_{R}\right)=\left(x_{R_{\left(\left(k_{R}\right)\right.}} k_{R} \oplus x_{R}\right)\left[1, \ldots, i\left(k_{R}\right)-1, i\left(k_{R}\right)+1, \ldots, n / 2\right] \tag{10}
\end{align*}
$$

where $i\left(k_{L}\right)$ and $i\left(k_{R}\right)$ denote the indices of the lowest bit which is set to 1 in $k_{L}, k_{R}$, respectively. Moreover, it is easy to deduce that $\Phi_{k_{L}}$ and $\Phi_{k_{R}}$ are both linear functions, $\operatorname{Ker} \Phi_{k_{L}}=\left\{\mathbf{0}, k_{L}\right\}$, and $\operatorname{Ker} \Phi_{k_{R}}=\left\{\mathbf{0}, k_{R}\right\}$. In particular, the notation

$$
\left(x_{i\left(k_{L}\right)} k_{L} \oplus x_{L}\right)\left[1, \ldots, i\left(k_{L}\right)-1, i\left(k_{L}\right)+1, \ldots, n / 2\right]
$$

refers to an $(n / 2-1)$-bit vector, which consists of the bits of $x_{i\left(k_{L}\right)} k_{L} \oplus x_{L}$ except for the $i\left(k_{L}\right)_{\text {th }}$ bit.

Theorem 8 For the round function $F_{k, w}(x)$ of DBISON, which is defined by (2) and (10), the entries of LAT of $F_{k, w}(x)$ are determined as:

1) $\operatorname{LAT}_{F_{k, w}}[a, b]=2^{n-1}$, if $b_{L} \bullet k_{R}=b_{R} \bullet k_{L}=0, a_{R}=b_{L}$ and $a_{L} \oplus b_{L} \oplus b_{R}=\mathbf{0}$.
2) $\mathrm{LAT}_{F_{k, w}}[a, b]= \pm 2^{(3 n / 2-1) / 2}$, if $b_{L} \bullet k_{R}=0, b_{R} \bullet k_{L}=1, a_{R}=b_{L}$ and $a_{L} \oplus b_{L} \oplus b_{R}=\mathbf{0}$.
3) $\operatorname{LAT}_{F_{k, w}}[a, b] \in\left(-2^{(3 n / 2-1) / 2}, 2^{(3 n / 2-1) / 2}\right)$, if $b_{L} \bullet k_{R}=1$ and $\left(a_{L} \oplus b_{L}\right) \bullet k_{R}=0$.
4) Otherwise, $\operatorname{LAT}_{F_{k, w}}[a, b]=0$.

Proof By Definition 1, it is clear that

$$
\begin{aligned}
& \operatorname{LAT}_{F_{k, w}}[a, b]:=\left|\left\{x \in F_{2}^{n} \mid a \bullet x \oplus b \bullet F_{k, w}(x)=0\right\}\right|-2^{n-1}=\frac{1}{2} W_{F_{k, w}}(a, b) . \\
& W_{F_{k, w}}(a, b):=\sum_{x \in F_{2}^{n}}(-1)^{a \bullet \times \oplus b \bullet F_{k, w}(x)} \\
& =\sum_{x \in F_{2}^{n}}(-1)^{a_{L} \cdot x_{L} \oplus a_{R} \cdot x_{R} \oplus b_{L}} \cdot\left(x_{L} \oplus x_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L} \oplus x_{R}\right)\right) k_{R}\right) \oplus b_{R} \cdot\left(x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right) \\
& =\sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \cdot k_{L}} \sum_{x_{R} \in F_{2}^{n_{2}}}(-1)^{\left(a_{R} \oplus b_{L}\right) \cdot \cdot_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right) \oplus \Phi_{k_{R}}\left(x_{R}\right)\right) b_{L} \cdot \dot{k}_{R}} .
\end{aligned}
$$

According to the value of $b_{L} \bullet k_{R}, W_{F_{k, w}}(a, b)$ can be calculated in the following cases.
Case 1. $b_{L} \bullet k_{R}=0$.
In this case, $W_{F_{k, w}}(a, b)=\sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \cdot k_{L}} \sum_{x_{R} \in F_{2}^{n / 2}}(-1)^{\left(a_{R} \oplus b_{L}\right) \cdot x_{R}}:=W_{1}$.
If $a_{R} \neq b_{L}$, then $\sum_{x_{R} \in F_{2}^{n / 2}}(-1)^{\left(a_{R} \oplus b_{L}\right) \cdot x_{R}}=0$, thus $W_{1}=0$.
If $a_{R}=b_{L}$, then $W_{1}=2^{n / 2} \sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \bullet k_{L}}$. On the one hand, if $b_{R} \bullet k_{L}=0$, then $W_{1}=2^{n / 2} \sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L}}=\left\{\begin{array}{ll}0, & \text { if } a_{L} \oplus b_{L} \oplus b_{R} \neq \mathbf{0}, \\ 2^{n}, & \text { if } a_{L} \oplus b_{L} \oplus b_{R}=\mathbf{0} .\end{array}\right.$ On the other hand, if
$b_{R} \bullet k_{L}=1$, then $W_{1}=2^{n / 2} \sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)}$. Assuming that $\Phi_{k_{L}}\left(x_{L}\right)=y_{L}$, using that $\Phi_{k_{L}}$ is linear and $\operatorname{Ker} \Phi_{k_{L}}=\left\{\mathbf{0}, \boldsymbol{k}_{L}\right\}$, we obtain

$$
\begin{aligned}
\sum_{x_{L} \in F_{2}^{n-2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)} & =\sum_{y_{L} \in F_{2}^{n-1}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot \dot{y}_{L}^{\prime} \oplus f_{L}\left(\left(w_{L} \oplus y_{L}\right)\right.}+\sum_{y_{L} \in F_{2}^{n-1}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot\left(y_{L}^{\prime} \oplus k_{L}\right) \oplus f_{L}\left(w_{L} \oplus y_{L}\right)} \\
& =\left[1+(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot k_{L}}\right] \sum_{y_{L} \in F_{2}^{n_{2}-1}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot \cdot_{L}^{\prime} \oplus f_{L}\left(w_{L} \oplus y_{L}\right)},
\end{aligned}
$$

where $y_{L}^{\prime}$ is the same as $y$ with an additional bit set to zero at position $i\left(k_{L}\right)$. Furthermore, if $\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \bullet k_{L}=1$, then $W_{1}=2^{n / 2} \times 0=0$. If $\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \bullet k_{L}=0$, then

$$
W_{1}=2^{n / 2+1} \sum_{y_{L} \in F_{2}^{n-2-1}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot y_{L}^{\prime} \oplus f_{L}\left(w_{L} \oplus y_{L}\right)}
$$

Let $w_{L} \oplus y_{L}=u_{L}$, and accordingly $W_{1}=2^{n / 2+1}(-1)^{\left(a_{L}^{\oplus} \oplus b_{L}^{n} \oplus b_{R}^{n}\right) \cdot w_{L}} \sum_{u_{L} \in F_{2}^{n 2-1}}(-1)^{\left(a_{L}^{\oplus} \oplus b_{L}^{\oplus} \oplus b_{R}^{n}\right) \cdot u_{L} \oplus f_{L}\left(u_{L}\right)}$, where $a_{L}^{\prime \prime}$ is an $(n / 2-1)$-dimensional vector obtained by removing the bit in position $i\left(k_{L}\right)$ of $a_{L}$. Since $f_{L}$ is a bent function, then $W_{1}=2^{n / 2+1}(-1)^{\left(a_{L}^{\prime \prime} \oplus b_{L}^{\prime} \oplus b_{R}^{\prime}\right) \cdot \omega_{L}}\left( \pm 2^{(n / 2-1) / 2}\right)= \pm 2^{(3 n / 2+1) / 2}$.

Case 2. $b_{L} \bullet k_{R}=1$.

$$
W_{F_{k, w}}(a, b)=\sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \cdot k_{L}} \sum_{x_{R} \in F_{2}^{n / 2}}(-1)^{\left(a_{R} \oplus b_{L}\right) \cdot x_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right) \oplus \Phi_{k_{R}}\left(x_{R}\right)\right)} .
$$

For any fixed $x_{L} \in F_{2}^{n / 2}$, it can be calculated that

$$
\sum_{x_{R} \in F_{2}^{n / 2}}(-1)^{\left(a_{R} \oplus b_{L}\right) \cdot x_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right) \oplus \Phi_{k_{R}}\left(x_{R}\right)\right)}= \begin{cases}0, & \text { if }\left(a_{R} \oplus b_{L}\right) \bullet k_{R}=1, \\ \pm(-1)^{\left(a_{R}^{n} \oplus b_{L}^{\prime \prime}\right) \cdot\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right)\right)} 2^{(n / 2+1) / 2}, & \text { if }\left(a_{R} \oplus b_{L}\right) \bullet k_{R}=0 .\end{cases}
$$

Thus, if $\left(a_{R} \oplus b_{L}\right) \bullet k_{R}=1$, then $W_{F_{k, w}}(a, b)=2^{n / 2} \times 0=0$. If $\left(a_{R} \oplus b_{L}\right) \bullet k_{R}=0$, then

$$
W_{F_{k, w}}(a, b)= \pm 2^{(n / 2+1) / 2} \sum_{x_{L} \in F_{2}^{n / 2}}(-1)^{\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \cdot x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \cdot \bullet_{L} \oplus\left(a_{R}^{*} \oplus b_{L}^{n}\right) \cdot\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right)\right)} .
$$

Thus, $-2^{(3 n / 2+1) / 2} \leq W_{F_{k, w}}(a, b) \leq 2^{(3 n / 2+1) / 2}$, where the equalities hold if and only if for all $x_{L} \in F_{2}^{n / 2}$, we have

$$
\left(a_{R}^{\prime \prime} \oplus b_{L}^{\prime \prime}\right) \bullet\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{L}\right)\right) \oplus\left(a_{L} \oplus b_{L} \oplus b_{R}\right) \bullet x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) b_{R} \bullet k_{L}=0 \text { or } 1 .
$$

The probability that these extreme cases occurring is very small, thus we can suppose $-2^{(3 n / 2+1) / 2}<W_{F_{k, w}}(a, b)<2^{(3 n / 2+1) / 2}$. $\#$

Theorem 9 For DBISON cipher, let its round function $F_{k, w}(x)$ be given by (2) and (10). If the number of rounds is $r=n / 2+3$, then we have MLP $<2^{-(n-1)}$ for $n>4$.
Proof Assume that there exists a nontrivial linear characteristic $\theta=\left(\theta_{0}, \theta_{1}, \ldots, \theta_{n / 2+3}\right)$. In particular, let the linear characteristic $\theta^{*}=\left(\theta_{0}, \theta_{1}, \ldots, \theta_{n / 2}\right)$ be such that $\operatorname{LP}\left(\theta_{i-1}, \theta_{i}\right)=1$, $i=1,2, \ldots, n / 2$. By Theorem 8, we have $\operatorname{LP}\left(\theta_{i-1}, \theta_{i}\right)=1$ if and only if $\theta_{i L} \bullet k_{i R}=\theta_{i R} \bullet k_{i L}=0$, $\theta_{i L}=\theta_{(i-1) R}$ and $\theta_{i R}=\theta_{(i-1) L} \oplus \theta_{(i-1) R}$. Note that there are two constraints (two-bit constraint conditions) for each round subkey, i.e. $\theta_{i L} \bullet k_{i R}=\theta_{i R} \bullet k_{i L}=0$. In this case, considering $n / 2$ rounds, the cardinality of a weak subkey set (satisfying the constraint conditions) should be
only $2^{n} \times 2^{-2 \times(n / 2)}=1$ on average. On the other hand, if there are $n / 2+3-n / 2=3$ rounds, then the linear characteristic $\theta^{*}=\left(\theta_{n / 2}, \theta_{n / 2+1}, \theta_{n / 2+2}, \theta_{n / 2+3}\right)$ exists with probability $\left[2^{-(n / 2-1)}\right]^{3}=2^{-(3 n / 2-3)}$. Therefore, MLP $<2^{-(n-1)}$ for $n>4$.

Remark 6. To resist algebraic attacks, the default round number should be at least $3 n$.

## 5. DBISON instances and implementation results

In this section, we discuss our implementation of DBISON encryption algorithm with input block size of 10 bits, where the generations of round keys, whitened keys and round constants are also specified. Similarly to the standard BISON encryption algorithm, the bent function used in this instance of DBISON is the quadratic function $f\left(X_{1}, X_{2}\right)=X_{1} \bullet X_{2}$, where $X_{i} \in F_{2}^{5}$. The differential uniformity and nonlinearity for round-reduced versions of DBISON consisting of 30 rounds (alternatively 10 or 20 rounds) and for different instances (specifying different secret keys via LFSRs) are given. The truth table of one particular instance and the intermediate values for 30 encryption rounds are given in Appendix A and B, respectively.

Assume that the input bit string for DBISON is $x=\left(x_{10}, x_{9}, \ldots, x_{1}\right)$, which is divided into two parts, i.e. $x_{L}=\left(x_{10}, x_{9}, \ldots, x_{6}\right)$ and $x_{R}=\left(x_{5}, x_{4}, \ldots, x_{1}\right)$. The first encryption round is described below.

- The encryption operation for the left branch includes the following five steps.

1) The left key $k_{L}$ is derived from the state of an LFSR, where the primitive polynomial used is $x^{5}+x^{2}+1$, and the initial state belongs to $F_{2}^{5} \backslash\{\mathbf{0}\}$.
2) $\Phi_{k_{L}}\left(x_{L}\right)=\left(x_{L_{\left(l_{L}\right)}} k_{L} \oplus x_{L}\right)\left[1, \ldots, i\left(k_{L}\right)-1, i\left(k_{L}\right)+1, \ldots, 5\right]$.
3) The left whitened key $w_{L}$ is derived from the state of another LFSR, where the primitive polynomial used is $x^{4}+x^{3}+1$, and the initial state is fixed by $(1,0,0,0)$. The round constant $C_{L}$ is derived from the state of the same LFSR, and the initial state is given by ( $0,0,0,1$ ).
4) $\Phi_{k_{L}}\left(x_{L}\right) \oplus w_{L} \oplus C_{L}=\left(y_{4}, y_{3}, y_{2}, y_{1}\right), f\left(y_{4}, y_{3}, y_{2}, y_{1}\right)=y_{4} y_{2} \oplus y_{3} y_{1} \oplus b_{L}$, and $b_{L}=0$ for the first $r / 2$ rounds, and $b_{L}=1$ for the remaining $r / 2$ rounds, where $r$ is the number of rounds.
5) The value of $x_{L} \oplus f\left(y_{4}, y_{3}, y_{2}, y_{1}\right) k_{L}$ is calculated.

- The encryption operation for the right branch contains the five portions below. In particular, the input string for the right branch is $x_{L} \oplus x_{R}$, denote it as $x_{R}^{\prime}$.

1) The right-hand part of the key $k_{R}$ is derived from the state of an LFSR, where the primitive polynomial used is given by $x^{5}+x^{3}+1$, and the initial state belongs to $F_{2}^{5} \backslash\{\boldsymbol{0}\}$.
2) $\Phi_{k_{R}}\left(x_{R}^{\prime}\right)=\left(x_{R_{\left(k_{R}\right)}^{\prime}}^{\prime} k_{R} \oplus x_{R}^{\prime}\right)\left[1, \ldots, i\left(k_{R}\right)-1, i\left(k_{R}\right)+1, \ldots, 5\right]$.
3) The right-hand part of the whitened key $w_{R}$ is derived from the state of another LFSR, the primitive polynomial used is given by $x^{4}+x+1$, and the initial state is fixed by $(1,0$, 0,1 ). The round constant $C_{R}$ is derived from the state of the same LFSR, and the initial state is fixed by $(0,0,0,1)$.
4) $\Phi_{k_{R}}\left(x_{R}^{\prime}\right) \oplus w_{R} \oplus C_{R}=\left(y_{4}^{\prime}, y_{3}^{\prime}, y_{2}^{\prime}, y_{1}^{\prime}\right), f\left(y_{4}^{\prime}, y_{3}^{\prime}, y_{2}^{\prime}, y_{1}^{\prime}\right)=y_{4}^{\prime} y_{2}^{\prime} \oplus y_{3}^{\prime} y_{1}^{\prime} \oplus b_{R}$, and $b_{R}=0$ for the
first $r / 2$ rounds and $b_{R}=1$ for the remaining $r / 2$ rounds, where $r$ is the number of rounds.
5) The value of $x_{R}^{\prime} \oplus f\left(y_{4}^{\prime}, y_{3}^{\prime}, y_{2}^{\prime}, y_{1}^{\prime}\right) k_{R}$ is calculated.

Finally, the output value of the first round is $\left(x_{R}^{\prime} \oplus f\left(y_{4}^{\prime}, y_{3}^{\prime}, y_{2}^{\prime}, y_{1}^{\prime}\right) k_{R}, x_{L} \oplus f\left(y_{4}, y_{3}, y_{2}, y_{1}\right) k_{L}\right)$. Similarly, in the second round, $k, w$ and $C$ are also derived from the states of the corresponding LFSRs in the next clock, and so on. More specifically, the initial state of the LFSR for deriving $k_{L}$ in the first encryption round is fixed to any value in $F_{2}^{5} \backslash\{\boldsymbol{0}\}$. On the other hand, the initial state of the LFSR for deriving $k_{R}$ in the first round, selects another value $k_{L}$ in $F_{2}^{5} \backslash\{\boldsymbol{0}\}$. This gives in total 930 instances (different keys) of DBISON which we have checked. The differential uniformities and nonlinearities of these instances for DBISON that implements 10, 20 and 30 encryption rounds are verified, respectively. These results are described in Fig. 2 and Fig. 3. In particular, the horizontal axis represents the value of the differential uniformity (nonlinearity), whereas the vertical axis is the number of instances whose differential uniformity (nonlinearity) is fixed.


Fig. 2(a). The differential uniformities of 10-round DBISON


Fig. 2(b). The nonlinearities of 10 -round DBISON
In Fig. 2, for DBISON consisting of 10 encryption rounds, the differential uniformity is mainly distributed among the values $12,14,16$ and 18 . Actually, these values have a percentage of approximately $92.26 \%$. On the other hand, the maximal nonlinearity that has been achieved in the simulations is 440 . Also, the nonlinearity in the range between 384 and 440 stands for the percentage of approximately $95.91 \%$. In fact, it means that these functions achieve relatively high nonlinearity. (note that the nonlinearity of bent functions is 496, and the nonlinearity of almost optimal functions is 480 when $n=10$.) Moreover, the best differential uniformity of these instances is 14 , and the nonlinearity is 440 , which is quite
close to the almost optimal functions. This illustrates that most of these DBISON instances have quite good differential uniformity and nonlinearity, though only 10 encryption rounds are considered.


Fig. 3(a). The differential uniformities of 30 -round DBISON


Fig. 3(b). The nonlinearities of 30 -round DBISON
Fig. 3(a) illustrates that the differential uniformity takes values 12 and 14 with the percentage of approximately $93.51 \%$, when the number of rounds is 30 . The nonlinearity distribution is given in Fig. 3(b) and the nonlinearities between 428 and 442 occur with the percentage of approximately $95.2 \%$. There exist many DBISON instances, using 30 rounds, whose differential uniformity equals 12 and having nonlinearity 442 . The truth table of one of these instances is given in Appendix A, whereas the test vectors for each round are provided in Appendix B.

In addition, the differential uniformities and nonlinearities of DBISON instances using 20 rounds can be found in Appendix C. Comparing the 20 -round and 30 -round results, it is clear that their performances are quite close (of course 30 -round DBISON is somewhat better). Of course, all DBISON instances are balanced bijections. Therefore, DBISON has quite good cryptographic performance.

Similarly to the encryption operation, the decryptions of left branch and right branch are also performed in parallel. More precisely, let $\tau_{L}\left(x_{L}\right)=x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}$, $\tau_{R}\left(x_{R}\right)=x_{R} \oplus f_{R}\left(w_{R} \oplus \Phi_{k_{R}}\left(x_{R}\right)\right) k_{R}, x_{L}, \quad x_{R} \in F_{2}^{n / 2}$. Then, $\tau_{L}$ and $\tau_{R}$ can be derived as below. For any $x_{L} \in F_{2}^{n / 2}$,

$$
\begin{aligned}
\tau_{L} \circ \tau_{L}\left(x_{L}\right) & =\tau_{L}\left(x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right) \\
& =x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}\right)\right) k_{L} .
\end{aligned}
$$

If $f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=0$, it is clear that $\tau_{L} \circ \tau_{L}\left(x_{L}\right)=x_{L}$. If $f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right)=1$, then we have

$$
\tau_{L} \circ \tau_{L}\left(x_{L}\right)=x_{L} \oplus k_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L} \oplus k_{L}\right)\right) k_{L}=x_{L} \oplus k_{L} \oplus f_{L}\left(w_{L} \oplus \Phi_{k_{L}}\left(x_{L}\right)\right) k_{L}=x_{L},
$$ because $\operatorname{Ker} \Phi_{k_{L}}=\left\{\mathbf{0}, k_{L}\right\}$. Thus, $\tau_{L}$ is involutory, and this also holds for $\tau_{R}$.

Note that the round function $F(x)$ of DBISON can be represented as $F(x)=\left(\tau_{R}\left(x_{L} \oplus x_{R}\right), \tau_{L}\left(x_{L}\right)\right)$. Then, the output of the left branch is $y_{L}=\tau_{R}\left(x_{L} \oplus x_{R}\right)$, and the output of the right branch is $y_{R}=\tau_{L}\left(x_{L}\right)$. Since both $\tau_{L}$ and $\tau_{R}$ are involutory, we have $x_{L}=\tau_{L}\left(y_{R}\right), x_{L} \oplus x_{R}=\tau_{R}\left(y_{L}\right)$, that is, $x_{R}=\tau_{R}\left(y_{L}\right) \oplus \tau_{L}\left(y_{R}\right)$. The round decryption function is $F^{-1}(y)=\left(\tau_{L}\left(y_{R}\right), \tau_{R}\left(y_{L}\right) \oplus \tau_{L}\left(y_{R}\right)\right)$, see Fig. 4. Therefore, the decryption process actually uses the reversed encryption round keys.


Fig. 4. The decryption round function $F^{-1}(y)$ of DBISON

## 6. Conclusion

In this paper, a new block cipher DBISON has been proposed, which employs double layers of a BISON-like construction. Compared to the original BISON cipher, DBISON divides the input into two halves and the nonlinear round function is computed in parallel, which results in a better performance in both software and hardware. Moreover, DBISON consisting of $3 n$ rounds is provably resistant against differential and linear attacks. More precisely, it is shown the MDP is $1 / 2^{n-1}$ for $n$ encryption rounds, and the MLP is strictly less than $1 / 2^{n-1}$ when $(n / 2+3)$ encryption rounds are used. Actually, if we select the data block size $n=258$, then both MDP and MLP of DBISON are very close to the ideal value.

## Appendix

A. The truth table of a permutation $F$ on $F_{2}^{10}$ of one DBISON instance given in hexadecimal format ( $r=30$, differential uniformity is 12 , nonlinearity is 442)

| 0E7 | 35A | 324 | 2E8 | 11B | 08A | 29A | 025 | 3 AB | 3BA | 3CE | 2C1 | 1A9 | 143 | 0F4 | 155 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 07D | 10A | 228 | 15C | 177 | 2FB | 081 | 2D4 | 28D | 0A8 | 010 | 088 | 35C | 152 | 142 | 1FA |
| 326 | 056 | 2CB | 1B7 | 310 | 1A4 | 0AD | 0FD | 11D | 218 | 29C | 186 | 175 | 1C8 | 257 | 0BB |
| 2C2 | 2BE | 377 | 208 | 0A6 | 115 | 189 | 057 | 092 | 291 | 1C5 | 238 | 1C1 | 104 | 3E8 | 0BD |
| 11C | 114 | 27B | 293 | 067 | 06D | 052 | 132 | 331 | 0AB | 27E | 16E | 3D0 | 194 | 26F | 122 |
| 1A0 | 12A | 109 | 1BD | 262 | 1B2 | 068 | 229 | 342 | 0D9 | 255 | 0AF | 3CF | 184 | 369 | 1F3 |
| 1C9 | 3D6 | 0E1 | 27D | 2D7 | 290 | 36F | 01E | 384 | 312 | 11F | 049 | 2A9 | 07A | 007 | 35E |
| 3AF | 0B2 | 36A | 008 | 0FF | 063 | 034 | 01C | 102 | 32B | 009 | 268 | 3D3 | 261 | 08E | 210 |
| 0DF | 339 | 3E6 | 026 | 17F | 19F | 371 | 0C1 | 20E | 1CB | 2A2 | 2AE | 045 | 069 | 370 | 287 |
| 289 | 080 | 11E | 380 | 0BC | 18B | 0CE | 2C7 | 2AC | 265 | 241 | 121 | 3E1 | 03A | 1F8 | 3 A 5 |
| 329 | 2A4 | 252 | OEE | 070 | 0D0 | 0E6 | 10C | 0B3 | 3EB | 14C | 3A2 | 316 | 38D | 118 | 1FF |
| 292 | 382 | $3 F 7$ | 03C | 27C | 06B | 23D | 283 | 22D | 375 | 2DF | 34A | 079 | 062 | 353 | 3BC |
| 0EB | 0F2 | 16B | 318 | 181 | 0E2 | 3ED | 120 | 090 | 37D | 0FC | 13E | 1 AB | 385 | 3B4 | 3B5 |
| 01F | 134 | 21C | 279 | 3E5 | 39A | 191 | 38B | 093 | 29D | 0E4 | 386 | 311 | 2ED | 31D | 376 |
| 006 | 248 | 065 | 2BF | 072 | 105 | 110 | 18D | 359 | 1A1 | 270 | 0EC | 395 | 0DA | 2FC | 0B6 |
| 13F | 2CD | 187 | 0D2 | 319 | 307 | 39C | 3E7 | 3B8 | 32C | 076 | 1A2 | 389 | 3FF | 226 | 1B0 |
| 2D9 | 2F8 | 18F | 12B | 309 | 28F | 15D | 17A | 251 | 3BD | 2DC | 3A8 | 123 | 213 | 05F | 2B9 |
| 0D1 | 31A | 39D | 22F | 18E | 1A5 | 38C | 3F4 | 235 | 346 | 373 | 0C5 | 335 | 089 | 1D8 | 1EB |
| 3C1 | 1B8 | 39F | 10B | 0BF | 024 | 29E | 394 | 095 | 09E | 2AB | 0C3 | 03E | 1DA | 042 | 02A |
| 3CA | 12E | 05C | 02B | 247 | 0СB | 023 | 0A9 | 1FD | 222 | 204 | 00A | 11A | 100 | 016 | 298 |
| 083 | 34B | 349 | 002 | 305 | 071 | 0F1 | 148 | 1 AC | 269 | 328 | 1E2 | 224 | 0C7 | 084 | 3F9 |
| 0E0 | 15A | 32A | 21A | 099 | 2BA | 07C | 147 | 16A | 219 | 1AF | 0F8 | 3DB | 2B2 | 321 | 091 |
| 356 | 202 | 1FE | 1F1 | 3 A 9 | 0FB | 237 | 392 | 25E | 2C6 | 0A7 | 05B | 207 | 2F6 | 2F7 | 157 |
| 308 | 200 | 1ED | 1A6 | 3A7 | 39E | 139 | 112 | 3AD | 1F4 | 3DA | 1C6 | 350 | 23B | 035 | 256 |
| 314 | 23A | 018 | 01A | 085 | 01D | 30C | 348 | 097 | 178 | 1CD | 399 | 2FA | 039 | 2F5 | 16F |
| 337 | 267 | 1F2 | 201 | 1D2 | 096 | 37E | 18C | 215 | 2D2 | 0ED | 082 | 203 | 153 | 15B | 0C2 |

B. Test vectors with intermediate results for the DBISON instance in Appendix A. The input value is 1000011001

| $i$ | $x_{L_{i}}$ | $k_{L_{i}}$ | $w_{L_{i}}$ | $C_{L_{i}}$ | $x_{R_{i}}$ | $k_{R_{i}}$ | $w_{R_{i}}$ | $C_{R_{i}}$ | $x_{L_{i+1}}$ | $x_{R_{i+1}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 10000 | 10110 | 0100 | 1000 | 11001 | 00110 | 0100 | 1000 | 01001 | 00110 |
| 1 | 01001 | 01011 | 0010 | 0100 | 00110 | 10011 | 0010 | 1100 | 0111 | 01001 |
| 2 | 01111 | 00101 | 1001 | 0010 | 01001 | 11001 | 0001 | 1110 | 00110 | 01010 |
| 3 | 00110 | 10010 | 1100 | 1001 | 01010 | 11100 | 1000 | 1111 | 10000 | 00110 |
| 4 | 10000 | 01001 | 0110 | 1100 | 00110 | 11110 | 1100 | 0111 | 10110 | 10000 |
| 5 | 10110 | 00100 | 1011 | 0110 | 10000 | 11111 | 1110 | 1011 | 00110 | 10010 |
| 6 | 00110 | 00010 | 0101 | 1011 | 10010 | 01111 | 1111 | 0101 | 10100 | 00110 |
| 7 | 10100 | 00001 | 1010 | 0101 | 00110 | 00111 | 0111 | 1010 | 10101 | 10101 |
| 8 | 10101 | 10000 | 1101 | 1010 | 10101 | 00011 | 1011 | 1101 | 00000 | 10101 |
| 9 | 00000 | 01000 | 1110 | 1101 | 10101 | 10001 | 0101 | 0110 | 00100 | 00000 |
| 10 | 00100 | 10100 | 1111 | 1110 | 00000 | 11000 | 1010 | 0011 | 11100 | 10000 |
| 11 | 11100 | 01010 | 0111 | 1111 | 10000 | 01100 | 1101 | 1001 | 01100 | 11100 |
| 12 | 01100 | 10101 | 0011 | 0111 | 11100 | 10110 | 0110 | 0100 | 10000 | 01100 |
| 13 | 10000 | 11010 | 0001 | 0011 | 01100 | 11011 | 0011 | 0010 | 11100 | 10000 |
| 14 | 11100 | 11101 | 1000 | 0001 | 10000 | 11101 | 1001 | 0001 | 01100 | 11100 |
| 15 | 01100 | 01110 | 0100 | 1000 | 11100 | 01110 | 0100 | 1000 | 11110 | 01100 |
| 16 | 11110 | 10111 | 0010 | 0100 | 01100 | 10111 | 0010 | 1100 | 10010 | 01001 |
| 17 | 10010 | 11011 | 1001 | 0010 | 01001 | 01011 | 0001 | 1110 | 11011 | 01001 |
| 18 | 11011 | 01101 | 1100 | 1001 | 01001 | 10101 | 1000 | 1111 | 00111 | 11011 |
| 19 | 00111 | 00110 | 0110 | 1100 | 11011 | 01010 | 1100 | 0111 | 11100 | 00111 |
| 20 | 11100 | 00011 | 1011 | 0110 | 00111 | 00101 | 1110 | 1011 | 11011 | 11111 |
| 21 | 11011 | 10001 | 0101 | 1011 | 11111 | 00010 | 1111 | 0101 | 00110 | 01010 |
| 22 | 00110 | 11000 | 1010 | 0101 | 01010 | 00001 | 0111 | 1010 | 01100 | 11110 |
| 23 | 01100 | 11100 | 1101 | 1010 | 11110 | 10000 | 1011 | 1101 | 00010 | 01100 |
| 24 | 00010 | 11110 | 1110 | 1101 | 01100 | 01000 | 0101 | 0110 | 01110 | 11100 |
| 25 | 01110 | 11111 | 1111 | 1110 | 11100 | 00100 | 1010 | 0011 | 10110 | 10001 |
| 26 | 10110 | 01111 | 0111 | 1111 | 10001 | 10010 | 1101 | 1001 | 10101 | 11001 |
| 27 | 10101 | 00111 | 0011 | 0111 | 11001 | 01001 | 0110 | 0100 | 01100 | 10101 |
| 28 | 01100 | 10011 | 0001 | 0011 | 10101 | 10100 | 0011 | 0010 | 01101 | 01101 |
| 29 | 01101 | 11001 | 1000 | 0001 | 01101 | 11010 | 1001 | 0001 | 10100 | 11011 |

C. Distribution of the differential uniformity and nonlinearity for 20-round DBISON

(a) Distribution of the differential uniformity for 20-round DBISON

(b) Distribution of the nonlinearity for 20-round DBISON

## References

[1] C. E. Shannon, "Communication theory of secrecy systems," Bell System Technical Journal, vol. 28, no. 4, pp. 656-715, 1949. Article (CrossRef Link)
[2] M. Kanda, "Practical security evaluation against differential and linear cryptanalyses for Feistel ciphers with SPN round function," in Proc. of SAC 2000: Selected Areas in Cryptography-SAC 2000, Ontario, Canada, pp. 324-338, 2000. Article (CrossRef Link)
[3] J. Zhang and W. L. Wu, "Authenticated encryption based on SM4 round function," Acta Electronica Sinica, vol. 46, no.6, pp. 1294-1299, 2018. Article (CrossRef Link)
[4] J. Daemen and V. Rijmen, The Design of Rijndael: AES - The Advanced Encryption Standard, Berlin, Germany: Springer, 2002. Article (CrossRef Link)
[5] M. Matsui, "New block encryption algorithm MISTY," in Proc. of FSE 1997: Fast Software Encryption-FSE'97, Haifa, Israel, pp. 54-68, 1997. Article (CrossRef Link)
[6] S. Vaudenay, "On the Lai-Massey scheme," in Proc. of Advances in Cryptology-ASIACRYPT'99, Singapore, pp. 8-19, 1999. Article (CrossRef Link)
[7] A. Hamza, D. Shehzad, M. S. Sarfraz, et al., "Novel secure hybrid image steganography technique based on pattern matching," KSII Transactions on Internet and Information Systems, vol. 15, no. 3, pp. 1051-1077, 2021. Article (CrossRef Link)
[8] J. Daemen and V. Rijmen, "Security of a wide trail design," in Proc. of CryptologyINDOCRYPT 2002, Hyderabad, India, pp. 1-11, 2002. Article (CrossRef Link)
[9] L. Grassi, C. Rechberger, and S. Rønjom, "Subspace trail cryptanalysis and its applications to AES," IACR Trans. Symm.Cryptol, vol. 2016, no. 2, pp. 192-225, 2017. Article (CrossRef Link)
[10] L. Grassi, C. Rechberger, and S. Rønjom, "A new structural-differential property of 5-round AES," in Proc. of EUROCRYPT 2017, Paris, France, pp. 289-317, 2017. Article (CrossRef Link)
[11] S. Tessaro, "Optimally secure block ciphers from ideal primitives," in Proc. of ASIACRYPT 2015, Auckland, New Zealand, pp. 437-462, 2015. Article (CrossRef Link)
[12] V. T. Hoang, B. Morris and P. Rogaway, "An enciphering scheme based on a card shuffle," in Proc. of CRYPTO 2012, California, USA, pp. 1-13, 2012. Article (CrossRef Link)
[13] S. Vaudenay, "The end of encryption based on card shuffling," in Proc. of CRYPTO 2012 Rump Session, California, USA, 2012. Article (CrossRef Link)
[14] A. Canteaut, V. Lallemand, G. Leander, et al., "BISON instantiating the Whitened Swap-Or-Not construction," in Proc. of EUROCRYPT 2019, Darmstadt, Germany, pp. 585-616, 2019.
Article (CrossRef Link)
[15] E. Biham and A. Shamir, "Differential cryptanalysis of DES-like cryptosystems," Journal of Cryptology, vol. 4, pp. 3-72, 1991. Article (CrossRef Link)
[16] T. Kranz, G. Leander and F. Wiemer, "Linear cryptanalysis: key schedules and tweakable block ciphers," IACR Trans. Symmetric Cryptol, vol. 2017, no. 1, pp. 474-505, 2017. Article (CrossRef Link)
[17] N. T. Courtois and G. V. Bard, "Algebraic cryptanalysis of the Data Encryption Standard," in Proc. of Cryptography and Coding 2007, Cirencester, UK, pp. 152-169, 2007. Article (CrossRef Link)
[18] A. Canteaut and J. Roué, "On the behaviors of affine equivalent S-boxes regarding differential and linear attacks," in Proc. of EUROCRYPT 2015, Sofia, Bulgaria, pp. 45-74, 2015. Article (CrossRef Link)
[19] C. Li, B Sun, R. Li, et al., Attack Methods and Instances Analysis for Block Ciphers, Beijing, China: Science Press, 2010.
[20] X. Lai, J. L. Massey and S. Murphy, "Markov ciphers and differential cryptanalysis," in Proc. of EUROCRYPT 1991, Brighton, UK, pp. 17-38, 1991. Article (CrossRef Link)


Haixia Zhao is a PhD student in information security at Key Laboratory of Cognitive Radio and Information Processing, Ministry of Education, Guilin University of Electronic Technology, Guilin, China. She has received an MS degree from Southwest University, Chongqing, China, in 2007. Her research interests include cryptographic functions and cryptanalysis of block ciphers.


Yongzhuang Wei is a professor at Guangxi Key Laboratory of Cryptography and Information Security, Guilin University of Electronic Technology, Guilin, China. He received an MS degree and PhD degree from Xidian University, Xian, China, in 2004 and 2009, respectively. His research interests include the design and analysis of symmetric encryption algorithms.


Zhenghong Liu is an associate professor at Key Laboratory of Cognitive Radio and Information Processing, Ministry of Education, Guilin University of Electronic Technology, Guilin, China. He has received an MS degree from Guilin University of Electronic Technology, Guilin, China, in 2009. His research interests include wideband signal processing, intelligence information process, and FPGA hardware design.


[^0]:    This research was supported by the Natural Science Foundation of China (61872103, 62162016, 62062026), the Guangxi Natural Science Foundation(2019GXNSFGA245004, 2020GXNSFAA159076), the Foundation of Key Laboratory of Cognitive Radio and Information Processing , Ministry of Education (CRKL180107)

